

# On divergent 3-vertices in noncommutative $SU(2)$ gauge theory

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## Abstract

We analyze divergencies in 2-point and 3-point functions for noncommutative  $\theta$ -expanded  $SU(2)$ -gauge theory with massless fermions. We show that, after field redefinition and renormalization of couplings, one divergent term remains.

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# 1 Introduction and notation

One of the main motives to study noncommutativity of the spatial coordinates is a belief that it will provide a mechanism for regularization of ultraviolet divergencies. There are various results which support this idea [1, 2]. However, in the field-theoretic models on noncommutative Minkowski space, obstructions to renormalizability appear, as for example the UV/IR mixing [3]. Recently some theories were proposed [4, 5] where renormalizability was restored by modifications of the original lagrangian.

One should note that the representation of noncommutative fields plays an important role in aforementioned considerations. Indeed, not every representation allows even a definition of the cyclic trace and thus of the action. In this letter we study noncommutative gauge theories in the so-called  $\theta$ -expanded representation or approach, as given by [6, 7]. There are various results regarding renormalizability in this approach; they could roughly be summarized as ‘almost renormalizability’. This means that, in all examples which were considered, only one divergency in the effective action, the four-fermion vertex, remained after a generalized renormalization procedure. This was our motivation for a further and more detailed analysis of the structure of divergencies; a part concerning the  $SU(2)$  gauge theory is presented here.

To start with, we recall necessary definitions and give a brief review of the results relevant for our analysis. We discuss gauge and matter fields on the noncommutative Minkowski space defined by the commutation relation between the coordinates  $\hat{x}^\mu$  ( $\mu = 0, 1, 2, 3$ ):

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} = \text{const.}$$

The fields in this case can be represented in the space of functions on  $\mathbf{R}^4$ ; the multiplication is given by the Moyal product:

$$\phi(x) \star \chi(x) = e^{\frac{i}{2} \theta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y^\nu}} \phi(x) \chi(y)|_{y \rightarrow x} . \quad (1.1)$$

The basic idea of the approach is that, in order to represent arbitrary gauge symmetries, one has to enlarge the corresponding algebra. The gauge fields thus take values in the enveloping algebra of the given Lie group; the vector potential  $\hat{A}_\rho$  can be expanded in the basis of the symmetrized products of generators  $T^a, : T^a T^b \dots :$ . An important fact is that the coefficients in this expansion are not independent fields; they are derivatives of the commutative gauge potential. Moreover, the expansion (known as the Seiberg-Witten (SW) expansion, [8]) coincides with the expansion in the parameter  $\theta^{\mu\nu}$ . To first order in  $\theta^{\mu\nu}$  it reads:

$$\begin{aligned} \hat{A}_\rho(x) &= A_\rho(x) - \frac{1}{4} \theta^{\mu\nu} \{A_\mu(x), \partial_\nu A_\rho(x) + F_{\nu\rho}(x)\} + \dots \\ \hat{\psi}(x) &= \psi(x) - \frac{1}{2} \theta^{\mu\nu} A_\mu(x) \partial_\nu \psi(x) + \frac{i}{4} \theta^{\mu\nu} A_\mu(x) A_\nu(x) \psi(x) + \dots \end{aligned} \quad (1.2)$$

We see that the noncommutative field expressed in terms of its commutative counterpart is nonlocal, as (1.2) has infinitely many terms. The SW expansion is not given uniquely, and this is an important fact which we will use later.

The (1.2) are general expressions, valid for any gauge group. Here we restrict our discussion to the  $SU(2)$  gauge theory with fermions in the fundamental representation; for the reasons which we explain shortly the fermions are massless. Inserting (1.2) in the noncommutative action, in linear order we obtain:

$$S = S_0 + S_{1,A} + S_{1,\psi}. \quad (1.3)$$

In the case of  $SU(2)$  and massless fermions (1.3) reads

$$S_0 = \int d^4x \left( \bar{\psi}(i\gamma^\mu D_\mu)\psi - \frac{1}{4}F^{\mu\nu a}F_{\mu\nu}^a \right), \quad (1.4)$$

$$S_{1,A} = 0,$$

$$\begin{aligned} S_{1,\psi} &= \frac{1}{2}\theta^{\rho\sigma} \int d^4x \left( -i\bar{\psi}\gamma^\mu F_{\mu\rho} D_\sigma\psi - \frac{i}{2}\bar{\psi}F_{\rho\sigma}\gamma^\mu D_\mu\psi \right) \\ &= -\frac{1}{8}\theta^{\rho\sigma}\Delta_{\rho\sigma}^{\mu\nu\alpha} \int d^4x \bar{\psi}F_{\mu\nu}\gamma^\beta(i\partial_\alpha + A_\alpha)\psi, \end{aligned} \quad (1.5)$$

where  $\Delta_{\sigma\rho\mu}^{\alpha\beta\gamma} = \delta_\sigma^\alpha\delta_\rho^\beta\delta_\mu^\gamma - \delta_\rho^\alpha\delta_\sigma^\beta\delta_\mu^\gamma + (\text{cyclic } \alpha\beta\gamma) = -\epsilon^{\alpha\beta\gamma\lambda}\epsilon_{\sigma\rho\mu\lambda}$ . In the general case the linear bosonic term  $S_{1,A}$  does not vanish; it depends on the symmetric symbols  $d_{abc}$  of the representation of the gauge potential [7]. The first order corrections  $S_{1,A}$  and  $S_{1,\psi}$  can be treated as new couplings which describe the effects of noncommutativity. In all orders, the correction terms are invariant to the commutative gauge transformations; they, however, explicitly depend on the choice of representation. One sees immediately that the action (1.4) has a smooth limit  $\theta \rightarrow 0$ ; in this limit it reduces to the ordinary gauge theory. The limit is physically very important as  $\theta$  is small, of the order of magnitude of  $l_{\text{Planck}}^2$ . Of course, the existence of a smooth limit could be considered as a drawback too, because in many noncommutative models the commutative limit is singular, for example in ordinary quantum mechanics.

The fact that the ‘coupling constant’  $\theta^{\mu\nu}$  is dimensionfull implies the apparent non-renormalizability of the theory, unless there is some additional symmetry. The idea that nonuniqueness of the expansion (1.2) might play the role of a symmetry appeared first in [9]. They found that the divergencies in the photon propagator in noncommutative electrodynamics can be, in  $\theta$ -linear order, absorbed in a redefinition of fields; moreover, that such redefinition can be generalized to all orders in  $\theta$ . Let us formulate this more precisely. Write the SW expansion as

$$\hat{A}_\mu = \sum A_\mu^{(n)}, \quad \hat{\psi} = \sum \psi^{(n)}, \quad (1.6)$$

where  $A_\mu^{(n)}, \psi^{(n)}$  denote the terms of the  $n$ -th order in  $\theta$ . Then the transformation

$$A_\mu^{(n)} \rightarrow A_\mu^{(n)} + \mathbf{A}_\mu^{(n)}, \quad \psi^{(n)} \rightarrow \psi^{(n)} + \Psi^{(n)}, \quad (1.7)$$

does not change the noncommutative fields  $\hat{A}_\mu, \hat{\psi}$  if  $\mathbf{A}_\mu^{(n)}, \Psi^{(n)}$  are gauge covariant expressions containing exactly  $n$  factors  $\theta$  [9, 10]. The change of the action induced by the shift (1.7) is

$$\begin{aligned} \Delta S^{(n,A)} &= \int d^4x (D_\nu F^{\mu\nu})\mathbf{A}_\mu^{(n)}, \\ \Delta S^{(n,\psi)} &= \int d^4x \left( \bar{\psi}i\not{D}\Psi^{(n)} + \bar{\Psi}^{(n)}i\not{D}\psi \right). \end{aligned} \quad (1.8)$$

Thus, along with  $S$  all actions  $S + \Delta S^{(n)}$  describe the same physical theory. One might conclude that, if divergent terms in the effective action are of the form (1.8), the theory is renormalizable. The divergencies can be absorbed by the SW redefinition: the new, redefined fields are the physical ones. This, in some sense, generalizes the usual notion of renormalizability and gives an additional freedom to the theory.

A more detailed analysis of noncommutative electrodynamics with fermions was done in [11] to first order in  $\theta$ . The conclusion was that all propagators and vertices, with the exception of the 4-fermion vertex, are renormalizable if fermions are massless. This result was confirmed to  $\theta^2$ -order for the propagators in [12].

## 2 Divergencies in $SU(2)$ and renormalization

The divergencies of the one-loop effective action for  $SU(2)$  gauge theory coupled to the massless fermions were calculated in linear order in [13]. The classical action of the theory is (1.4-1.5); details about the quantization, gauge fixing, ghosts etc. are explained in [13]. In the zero-th order, the divergent part of the one-loop effective action is given by

$$\Gamma_2 = \frac{1}{(4\pi)^2\epsilon} \int d^4x \left( \frac{10}{3} F_{\mu\nu}^a F^{\mu\nu a} + \frac{3i}{2} \bar{\psi} D \psi \right). \quad (2.1)$$

Here and below the divergencies in the  $n$ -point functions are expressed in the ‘covariantized’ form. This means that for example, the covariant 2-point function contains parts of the higher-point functions which are necessary to obtain the covariant instead of the partial derivatives. The  $\theta$ -linear divergent 2-point function is

$$\Gamma'_2 = \frac{1}{(4\pi)^2\epsilon} \frac{i}{8} \theta^{\mu\nu} \epsilon_{\mu\nu\rho\sigma} \int d^4x \bar{\psi} \gamma_5 \gamma^\sigma D^2 D^\rho \psi, \quad (2.2)$$

whereas the divergence in the 3-point function reads

$$\begin{aligned} \Gamma'_3 = & -\frac{1}{(4\pi)^2\epsilon} \frac{1}{2} \theta^{\mu\nu} \int d^4x \left( -\frac{37i}{4} \bar{\psi} \gamma^\alpha (F_{\nu\alpha} D_\mu + F_{\mu\nu} D_\alpha + F_{\alpha\mu} D_\nu) \psi \right. \\ & - \frac{6i}{4} \bar{\psi} \gamma^\alpha F_{\mu\nu} D_\alpha \psi - \frac{3i}{4} \bar{\psi} \gamma^\alpha (D_\alpha F_{\mu\nu}) \psi \\ & + 2i \bar{\psi} \gamma_\mu F_{\nu\alpha} D^\alpha \psi + i \bar{\psi} \gamma_\mu (D^\alpha F_{\nu\alpha}) \psi \\ & + \frac{5}{8} \epsilon_{\nu\alpha\beta\rho} \bar{\psi} \gamma_5 \gamma^\rho (D_\mu F^{\alpha\beta}) \psi - \frac{1}{16} \epsilon_{\mu\nu\alpha\beta} \bar{\psi} \gamma_5 \gamma^\sigma (D_\sigma F^{\alpha\beta}) \psi \\ & \left. + \frac{1}{8} \epsilon_{\mu\nu\alpha\beta} (2 \bar{\psi} \gamma_5 \gamma^\beta F^{\rho\alpha} D_\rho \psi + \bar{\psi} \gamma_5 \gamma^\beta (D_\rho F^{\rho\alpha}) \psi) \right). \end{aligned} \quad (2.3)$$

The divergent 4-fermion vertex has the same form as in  $U(1)$  theory:

$$\Gamma'_{4\psi} = \frac{1}{(4\pi)^2\epsilon} \frac{9}{32} \theta^{\mu\nu} \epsilon_{\mu\nu\rho\sigma} \int d^4x \bar{\psi} \gamma_5 \gamma^\sigma \psi \bar{\psi} \gamma^\rho \psi. \quad (2.4)$$

Apparently,  $\Gamma'_2$  and  $\Gamma'_3$  are of the form given by (1.8). Therefore we could conclude that, as in  $U(1)$ , the only obstacle to renormalization is the term (2.4). However, to prove this we have to construct the field redefinition which removes the divergencies explicitly. From the forms (2.2-2.3) we see that only the fermionic field needs to be redefined. The most general redefinition (1.7) in the first order in  $\theta$  is

$$\Psi^{(1)} = \theta^{\mu\nu} \left( \kappa_1 F_{\mu\nu} + i\kappa_2 \sigma_{\mu\rho} F_\nu{}^\rho + i\kappa_3 \epsilon_{\mu\nu\rho\sigma} \gamma_5 F^{\rho\sigma} + \kappa_4 \sigma_{\mu\nu} D^2 + \kappa_5 \sigma_{\rho\mu} D_\nu D^\rho \right) \psi, \quad (2.5)$$

and has 5 free parameters,  $\kappa_1, \dots, \kappa_5$ . The corresponding change in the lagrangian (that is, its  $\theta$ -linear part) reads:

$$\begin{aligned} \Delta\mathcal{L}^{(1)} = & i\theta^{\alpha\beta} \left( \left( \kappa_1 - \frac{1}{2} \kappa_5 \right) (\bar{\psi} \gamma^\mu (D_\mu F_{\alpha\beta}) \psi + 2 \bar{\psi} \gamma^\mu F_{\alpha\beta} D_\mu \psi) \right. \\ & - \kappa_2 (\bar{\psi} \gamma^\rho (D_\alpha F_{\beta\rho}) \psi + 2 \bar{\psi} \gamma^\rho F_{\beta\rho} D_\alpha \psi) \\ & + \kappa_2 (\bar{\psi} \gamma_\alpha (D_\mu F_\beta{}^\mu) \psi + 2 \bar{\psi} \gamma_\alpha F_\beta{}^\mu D_\mu \psi) \\ & + i\kappa_2 \epsilon_{\mu\alpha\rho\sigma} \bar{\psi} \gamma_5 \gamma^\sigma (D^\mu F_\beta{}^\rho) \psi - i\kappa_3 \epsilon_{\alpha\beta\rho\sigma} \bar{\psi} \gamma_5 \gamma^\mu (D_\mu F_{\rho\sigma}) \psi \\ & + \kappa_4 \epsilon_{\rho\alpha\beta\sigma} \bar{\psi} \gamma_5 \gamma^\sigma (2 D^2 D^\rho \psi - 2i F^{\rho\mu} D_\mu \psi - i (D_\mu F^{\mu\rho}) \psi) \\ & \left. + 2\kappa_4 (\bar{\psi} \gamma_\beta (D_\mu F_\alpha{}^\mu) \psi + 2 \bar{\psi} \gamma_\beta F_{\alpha\mu} D^\mu \psi) \right). \end{aligned} \quad (2.6)$$

Note that, as  $\kappa_1$  and  $\kappa_5$  appear only in the combination  $\kappa_1 - \kappa_5/2$ , one of those two parameters is superfluous; we take  $\kappa_5 = 0$ . After the shift  $\psi \rightarrow \psi + \Psi^{(1)}$  the renormalized Lagrangian becomes

$$\begin{aligned}
\mathcal{L} + \mathcal{L}_{ct} + \Delta\mathcal{L}^{(1)} = & -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} \left(1 + \frac{40g^2}{3(4\pi)^2\epsilon}\right) \\
& + i\bar{\psi}\not{\partial}\psi \left(1 - \frac{3g^2}{2(4\pi)^2\epsilon}\right) + g\mu^{\frac{\epsilon}{2}}\bar{\psi}A_\mu^a\gamma^\mu\frac{\sigma^a}{2}\psi \left(1 - \frac{3g^2}{2(4\pi)^2\epsilon}\right) \\
& + ig\mu^{\frac{\epsilon}{2}}\theta^{\alpha\beta}\bar{\psi}\gamma^\mu(F_{\beta\mu}D_\alpha + F_{\alpha\beta}D_\mu + F_{\mu\alpha}D_\beta)\psi \left(-\frac{1}{4} - \kappa_2 - \frac{37g^2}{8(4\pi)^2\epsilon}\right) \\
& + ig\mu^{\frac{\epsilon}{2}}\theta^{\alpha\beta}(\bar{\psi}\gamma^\mu(D_\mu F_{\alpha\beta})\psi + 2\bar{\psi}\gamma^\mu F_{\alpha\beta}D_\mu\psi) \left(\kappa_1 + \frac{\kappa_2}{2} - \frac{3g^2}{8(4\pi)^2\epsilon}\right) \\
& + ig\mu^{\frac{\epsilon}{2}}\theta^{\alpha\beta}(\bar{\psi}\gamma_\alpha(D^\mu F_{\beta\mu})\psi + 2\bar{\psi}\gamma_\alpha F_{\beta\mu}D^\mu\psi) \left(\kappa_2 - 2\kappa_4 + \frac{g^2}{2(4\pi)^2\epsilon}\right) \\
& - g\mu^{\frac{\epsilon}{2}}\theta^{\alpha\beta}\epsilon_{\beta\mu\nu\rho}\bar{\psi}\gamma_5\gamma^\rho(D_\alpha F^{\mu\nu})\psi \left(\frac{\kappa_2}{2} - \frac{5g^2}{16(4\pi)^2\epsilon}\right) \\
& + g\mu^{\frac{\epsilon}{2}}\theta^{\alpha\beta}\epsilon_{\alpha\beta\rho\sigma}\bar{\psi}\gamma_5\gamma^\mu(D_\mu F^{\rho\sigma})\psi \left(\kappa_3 - \frac{g^2}{32(4\pi)^2\epsilon}\right) \\
& + g\mu^{\frac{\epsilon}{2}}\theta^{\alpha\beta}\epsilon_{\alpha\beta\rho\sigma}(\bar{\psi}\gamma_5\gamma^\sigma(D_\mu F^{\rho\mu})\psi + 2\bar{\psi}\gamma_5\gamma^\sigma F^{\rho\mu}D_\mu\psi) \left(\kappa_4 - \frac{g^2}{16(4\pi)^2\epsilon}\right) \\
& - g\mu^{\frac{\epsilon}{2}}\theta^{\alpha\beta}\epsilon_{\alpha\beta\rho\sigma}\bar{\psi}\gamma_5\gamma^\sigma D^2 D^\rho\psi \left(2\kappa_4 - \frac{g^2}{8(4\pi)^2\epsilon}\right).
\end{aligned} \tag{2.7}$$

If we choose

$$\kappa_1 = \frac{g}{16(4\pi)^2\epsilon}, \quad \kappa_2 = \frac{5g}{8(4\pi)^2\epsilon}, \quad \kappa_3 = \frac{g}{32(4\pi)^2\epsilon}, \quad \kappa_4 = \frac{g}{16(4\pi)^2\epsilon},$$

the expression (2.7) reduces to

$$\begin{aligned}
\mathcal{L} + \mathcal{L}_{ct} + \Delta\mathcal{L}^{(1)} = & -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} \left(1 + \frac{40g^2}{3(4\pi)^2\epsilon}\right) \\
& + i\bar{\psi}\not{\partial}\psi \left(1 - \frac{3g^2}{2(4\pi)^2\epsilon}\right) + g\mu^{\frac{\epsilon}{2}}\bar{\psi}A_\mu^a\gamma^\mu\frac{\sigma^a}{2}\psi \left(1 - \frac{3g^2}{2(4\pi)^2\epsilon}\right) \\
& - \frac{i}{4}g\mu^{\frac{\epsilon}{2}}\theta^{\alpha\beta}\bar{\psi}\gamma^\mu(F_{\beta\mu}D_\alpha + F_{\alpha\beta}D_\mu + F_{\mu\alpha}D_\beta)\psi \left(1 + \frac{21g^2}{(4\pi)^2\epsilon}\right) \\
& + ig\mu^{\frac{\epsilon}{2}}\theta^{\alpha\beta}(\bar{\psi}\gamma_\alpha(D^\mu F_{\beta\mu})\psi + 2\bar{\psi}\gamma_\alpha F_{\beta\mu}D^\mu\psi) \frac{g^2}{(4\pi)^2\epsilon}.
\end{aligned} \tag{2.8}$$

Now, one would introduce the bare fields and the couplings as

$$\begin{aligned}
\psi_0 &= \sqrt{Z_2}\psi = \sqrt{1 - \frac{3g^2}{2(4\pi)^2\epsilon}}\psi, \\
A_0^\mu &= \sqrt{Z_3}A^\mu = \sqrt{1 + \frac{40g^2}{3(4\pi)^2\epsilon}}A^\mu, \\
g_0 &= g\mu^{\frac{\epsilon}{2}}Z_3^{-1/2}Z_2^{-1}\left(1 - \frac{3g^2}{2(4\pi)^2\epsilon}\right) = g\mu^{\frac{\epsilon}{2}}\left(1 - \frac{20g^2}{3(4\pi)^2\epsilon}\right) \\
\theta_0^{\alpha\beta} &= \theta^{\alpha\beta}Z_2^{-1}Z_3^{-\frac{1}{2}}\left(1 + \frac{21g^2}{(4\pi)^2\epsilon}\right)\left(1 - \frac{20g^2}{3(4\pi)^2\epsilon}\right)^{-1} = \theta^{\alpha\beta}\left(1 + \frac{45g^2}{2(4\pi)^2\epsilon}\right),
\end{aligned} \tag{2.9}$$

were the last term in (2.8) absent. Then the renormalized Lagrangian would be

$$\begin{aligned}
\mathcal{L} + \mathcal{L}_{ct} + \Delta\mathcal{L}^{(1)} = & -\frac{1}{4}F_{\mu\nu 0}^a F_0^{\mu\nu a} + i\bar{\psi}_0 \not{D}\psi_0 + g_0\bar{\psi}_0\gamma_\mu A_0^{\mu a}\frac{\sigma^a}{2}\psi_0 \\
& -\frac{i}{4}g_0\theta_0^{\alpha\beta}\bar{\psi}_0(F_{\beta\mu 0}D_\alpha + F_{\alpha\beta 0}D_\mu + F_{\mu\alpha 0}D_\beta)\psi_0 \\
& + ig\mu^{\frac{\epsilon}{2}}\theta^{\alpha\beta}(\bar{\psi}\gamma_\alpha(D^\mu F_{\beta\mu})\psi + 2\bar{\psi}\gamma_\alpha F_{\beta\mu}D^\mu\psi)\frac{g^2}{(4\pi)^2\epsilon}.
\end{aligned} \tag{2.10}$$

Unfortunately, the last term, which is divergent, remains: it cannot be absorbed by SW the redefinition or by the renormalization procedure.

### 3 Conclusion

As we saw, the SW redefinition does not possess enough free parameters to absorb the divergencies of the 3-point functions. Thus we come to the conclusion: the  $\theta$ -expanded  $SU(2)$  theory is not renormalizable even for massless fermions. Apart from the 4-fermion vertex, 3-vertices also cannot be renormalized. This result differs from the corresponding one for noncommutative  $U(1)$ , [11]. Comparing the two, we may add that our technique is slightly simpler: we introduce the SW redefinition after the quantization and not already in the classical action. This however does not change the reasoning essentially.

The negative result about renormalizability which we presented is conclusive, as clearly it cannot be altered in higher orders in  $\theta$ . The presence of fermions prevents renormalizability, except of course in the supersymmetric theories. The behavior of the pure gauge theories on the other hand is still not fully clear and deserves further investigation.

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